Math 201–Exam 1 (Spring 19) Profs. R. Aoun, B. Shayya, and T. Tlas

- Please write your **section number** on your booklet.
- Please place your student **ID card** on the desk in front of you.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- **Unjustified** answers will receive little or no credit.

Problem 1 (answer on pages 1 and 2 of the booklet.)
(8 pts each) Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence.

(i) $a_n = (n+1)^{1/n}$ (ii) $b_n = \dfrac{3^n - 2^n}{n! + n^n}$ (iii) $c_n = \left(1 + \sin\dfrac{1}{n}\right)^n$

Problem 2 (answer on pages 3, 4, and 5 of the booklet.)
(8 pts each) Which of the following series converge, and which diverge? When possible, find the sum of the series.

(a) $\sum_{n=0}^{\infty}\left(\frac{(-3)^{n+1}}{5^{n-1}} + \frac{(-2)^n}{3^{n+2}}\right)$ (b) $\sum_{n=1}^{\infty}\frac{(2n)!}{(2^n)(n!)(n!)}$

(c) $\sum_{n=1}^{\infty}(-1)^n\frac{e^{\cos n}}{n^3}$ (d) $\sum_{n=1}^{\infty}\left(8^{1/n} - 8^{1/(n+1)}\right)$

Problem 3 (answer on pages 6 and 7 of the booklet.)
Consider the power series

$$F(x) = \sum_{n=1}^{\infty}\frac{\ln n}{n^{1.5}}(x-3)^n.$$

(i) (16 pts) Find the radius and interval of convergence of $F(x)$.
(ii) (8 pts) Use the alternating series estimation theorem (ASET) to estimate $F(2)$ with an error of magnitude less than 0.25. Is your answer an over-estimate or an under-estimate? (Hint: $\ln 4 = 2\ln 2$.)

Problem 4 (answer on pages 8 and 9 of the booklet.)
(i) (12 pts) Use the integral test to prove that the p-series

$$\sum_{n=1}^{\infty}\frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

diverges if $0 < p \leq 1$, and converges if $p > 1$.
(ii) (8 pts) Prove that

$$\lim_{p\to\infty}\left(\sum_{n=1}^{\infty}\frac{1}{n^p}\right) = 1.$$